Key Stage 1:

- The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources (for example, concrete objects and measuring tools).
- By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
Year 1		Count on and back in 2, 5	Groups of	Solve simple one-step problems that involve using
		and 10.	Array	concrete objects and pictorial representations.
			Counting in	
			Sharing	
			Double	
			Half	
			Quarter	
Year 2	Identifying odd and even	Count in steps of 3 from 0	Odd, even,	Solve one-step problems involving multiplication
	numbers.	and in tens from any	Repeated	and division, using materials, arrays, repeated
	Recall multiples of 2, 5 and 10	number, forward or	addition/subtraction	addition, mental methods, and multiplication and
	and related division facts.	backward.	Grouping	division facts, including problems in contexts.
			Inverse	
			Multiply Multiple(s) of	They connect the 10 multiplication table to place
			Divide	value, and the 5 multiplication table to the divisions
			Division	on the clock face.
			Commutative	
			Calculate	
			Equivalent	

Year 1	Concrete	Pictorial	Conceptual
Multiplication and division as repeated addition and subtraction	Using familiar objects and resources. Finding 'groups of' with repeated addition and subtraction.	Repeated images E.g. How many legs?	
Represent repeated addition as an array. Begin to use arrays to find repeated subtraction.	Make arrays on grids with counting objects	Understand visual representations of arrays	
Doubling and halving numbers within 20 (as repeated addition and subtraction).	Using familiar objects and resources.	Using a variety of models and images.	Using number sentences and beginning to calculate mentally. $6+6=$ Double $9=$ $14=$ Double Half of $18=$ $\frac{1}{2}$ of $\square=5$ $10=$ half of $7=$ $14 \square$ $4+$ $\square=$ 8

Year 1	Concrete	Pictorial	Conceptual	Using and applying
Find and	Using familiar objects and	Using a variety of models and	Recognise unit fraction	I had 8 balloons.
name a		images _	notation:	I gave ¼ of them away.
half as one				How many balloons did I give
of two			½ and ¼	away?
equal	resources.			How many do I have left?
parts of a	resources.			
quantity.				Use the numbers 1 to 20.
				Which numbers can you find
Find and	in quarters			½ / ¼ of?
name a				What do you notice about your
quarter as	→			answers?
one of				
four equal				
parts of a	1 2 3 4 5 6 7 8 9 10 11 12 13			
quantity.				

Year 2	Concrete	Pictorial	Conceptual	Using and applying
Use arrays to make or draw multiplications and find the corresponding division facts. Write calculations using the multiplication (×) division (÷) and equals (=) signs.	Make arrays on grids using counting objects. Identify arrays in everyday objects.	Array images Repeated addition and subtraction along a number line.	Using number sentences and beginning to calculate mentally. $3 \times 4 = 12$ $12 \div 4 = 3$ Missing number problems. $20 = \square \times 5$ $3 = \square \div 6$	I had 20 lollies. I put them into groups of 5. How many groups were there? I had 20 lollies. I shared them between 5 people. How many lollies did each person get? I saved 5p each week for 6 weeks. How much did I save altogether?
Use arrays to understand the commutative law of multiplication.	Make arrays on grids using counting objects. 2 x 4 = 8 Rotating arrays to find other multiplications.	Repeated addition and subtraction along a number line. $ \begin{array}{cccccccccccccccccccccccccccccccccc$	Using number sentences and beginning to calculate mentally. $3 \times 5 = 15$ $5 \times 3 = 15$ $15 \div 5 = 3$ $15 \div 3 = 5$	If I save 5p each week, how many weeks will it take me to save 40p? There are 24 parents coming to watch our class assembly. How many different ways can you arrange the chairs? (In equal rows).

Lower Key Stage 2:

- The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.
- At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
Year 3	Recall multiples of 2, 5 and 10 and related division facts. Begin to recall multiples of 3, 6 and 4 and 8 and related division facts.	Count from 0 in multiples of 4, 8, 50 and 100.	Grid method Product Short division Remainder	Pupils should solve simple problems in contexts, including missing number problems, deciding which of the four operations to use and why, including measuring and scaling contexts, and correspondence problems in which m objects are connected to n objects (e.g. 3 hats and 4 coats, how many different outfits; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
Year 4	Recall multiplication and division facts for multiplication tables up to 12 × 12	Count in multiples of 6, 7, 9, 25 and 1 000 Multiply 3 numbers U x U x U Recall factor pairs for a given number	Factor Factor pair Quotient Divisor	Pupils should solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Year 3	Concrete	Pictorial	Conceptual	Using & applying
Year 3 TU x U Extending understanding of arrays.	Using counting objects and resources. 23 x 4 = 92 Tens Units	Pictorial Use arrays to link to grid multiplications. $ \begin{array}{c cccc} x & 20 & 3 \\ \hline 4 & 80 & 12 \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Conceptual Using number sentences. $18 \times 4 = \square$ $31 \times 3 = \square$ Missing number problems. $\square \times 41 = 123$ $7 \times \square = 84$	Year 3 went on a trip. There were 6 groups with 14 children in each group. How many children went on the trip in total? Use the digits 2, 3, 4, 5 and 6. Make a multiplication (U x TU) e.g. 2 x 53 = Find different totals can you find? How many multiplications
	Tens Units Output Discrete statements of the statement			have the <i>same</i> total?

Year 3	Concrete	Pictorial	Conceptual	Using & applying
TU ÷ U Sharing and grouping to create an array. (Not exchanging from tens to units at this stage).	Using counting objects and resources. $69 \div 3 = 23$ $10 10 1 1$ $10 10 1$ $10 10 1 1$	Extending divisions to resemble written method of short division. 23 3 9 9 1 1 69 ÷ 3 = 23 Check using multiplication inverse: 23 x 3 = 69	Short division. $ \begin{array}{c c} 23 \\ 3 69 \end{array} $ Check using multiplication inverse: $ \begin{array}{c c} x & 20 & 3 \\ \hline 3 & 60 & 9 \\ 60 + 9 = 69 \end{array} $	69 children were grouped equally onto 3 buses for a trip. How many children went on each bus? 3 children shared £69 equally. How much did they each receive? How many different divisions can you make? 36 ÷? =?
Understand the concept of remainders after division.	Using resources. 23 ÷ 4 = 5 r3	Repeated addition and subtractional along a number line. $23 \div 4 = 5 \text{ r}$ $\begin{array}{cccccccccccccccccccccccccccccccccccc$	Begin to solve mentally. $ \begin{array}{c} 23 \div 4 = \square \\ 31 \div 6 = \square \end{array} $ Missing number problems. $ \square \div 3 = 4^{r1} $ $ 17 \div \square = 3^{r2} $	A farmer had 33 eggs. He put them into boxes of 6. How many full boxes did he have? How many eggs did he have left over? If he put them into boxes of 12, how many would be left over now? Use each number in the 4x table. Make it with counters then share it into 3 groups. Write the remainder each time. What patterns do you notice? There are 75 children at sports day. If they were divided into 3 teams, 4 teams, 5 teams etc. how many children would be left out each time?

Year 4	Concrete	Pictorial	Conceptual	Using and applying
(Consolidate TU x U and extend to 3 digit numbers)	Crossing <i>one</i> boundary. 126 x 3 = Hundreds Tens Units	Beginning with grid multiplication.	Compact vertical method. 1 2 6 $\frac{x}{3}$ $\frac{3}{1}$ $\frac{7}{8}$	In one week, 163 people visited the museum each day. How many people visited in total? My sister and I were raising money for charity. We collected £127 every day for 6 days. We shared the money equally between two different charities. How much money did each charity receive?
	Extending to crossing <i>two</i> boundaries. 247 x 3 = Hundreds Tens Units	Beginning with grid multiplication.	Compact vertical method. $\begin{array}{cccc} 2 & 4 & 7 \\ \underline{x} & & 3 \\ \underline{7 & 4 & 1} \\ 1 & 2 \end{array}$	Use the digits 1, 2, 3 and 5. Make a multiplication U x HTU. How many different products are there? What are the largest and smallest products possible? U x HTU = 820. How many ways can you solve this?

Year 4 (Concrete	Pictorial	Conceptual	Using and applying
TU ÷ U (Where exchanging is required)	Grouping and sharing using place value counters. Exchanging counters which cannot be grouped. 138 ÷ 6 = 23 Hundreds Tens Units	Result of grouping/sharing counters during 'concrete' stage. 23 6 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Short division methods. 0 2 3 6 1 3 8 138 ÷ 6 = 23 432 ÷ 5 becomes 8 6 r 2 5 4 3 2 Answer: 86 remainder 2	A school ordered 432 pencils. They were put into packs of 5. How many packs were made? How many pencils were left over? Robbie has 150 stickers. He kept 12 and shared the rest equally between 6 friends. How many stickers did each of his friends get? 436 children need to be put into teams for sports day. How many different ways could the children be grouped equally? How many divisions can you make which have a remainder of 3? What patterns do you notice? Which numbers between 100 and 150 have a remainder of 1 when they are divided by 2, 3, 4, 5, and 6?

Upper Key Stage 2:

- The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.
- At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.
- By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

End of year expectations	Rapid recall	Mental calculation	Language	Using and applying
Year 5	Related decimal facts for tables E.g. 6 x 7 = 42 0.6 x 7 = 0.7 x 6 = 4.2÷7= etc.	Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000. ÷x 10, 100 or 1000 including decimals	Prime number Composite number Common factors Square / cube numbers	Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
Year 6		Perform mental calculations, including with mixed operations and large numbers E.g. 3 x 700 + 115 = Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.		Use their knowledge of the order of operations to carry out calculations involving the four operations. Solve addition and subtraction multi-step. Problems in contexts, deciding which operations and methods to use and why. Solve problems involving addition, subtraction, multiplication and division including interpreting remainders appropriately within the context of the problem.

Year 5	Concrete	Pictorial	Conceptual	Using and applying
Consolidate Y4 Formal vertical method HTU x U Lead to: ThHTU x U	Refer to Year 4 (HTU x U) and extend the process of using place value counters to ThHTU x U	Refer to Year 4 grid method (HTU x U) and extend to ThHTU x U	Refer to Year 4 vertical method (HTU x U) and extend to ThHTU x U $ \begin{array}{ccc} 2 & 7 & 4 & 1 \\ & \underline{x} & \underline{6} \\ & \underline{16446} \\ 4 & 2 \end{array} $ Answer: 2741 x 6 = 16446	There are 5 kittens, each weighing 1352g. What is their total mass in Kg? Use the digits 1 to 5. Make a multiplication: ThHTU x U How many products can you make between 5000 and 5500?
TU x TU	NB: Children working at this level are expected to have secured the use of place value counters to multiply by a one digit number. Therefore, they should proceed to pictorial methods.	Grid method: $47 \times 36 =$ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Compact method: 47	I saved £36 every week for a year. At the end of the year, I gave half of it to charity. How much money did I donate? Try this with several numbers: choose a prime number greater than 3, square it and divide the answer by 12. Look at the remainder. What do you notice? Why does this happen?

Year 5	Concrete	Pictorial	Conceptual	Using and applying
HTU x TU	Follow processes shown above (TU x TU).	Grid method leading to expanded vertical.	Compact method: $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	There are 24 bottles in a crate. Each bottle has a capacity of 720ml. what is the total amount in litres? Make 5 different 2 digit numbers e.g. 56, 74, 31, 65, 83. Multiply them each by 101. What do you notice? What happens when you multiply each one by 1001?
ThHTU ÷ U (Displaying remainders in different forms)	Follow processes shown in Year 4 HTU ÷ U with place value counters	Showing remainder as a whole number $858 \text{ r} 2$ $3 2^2 5^1 7^2 6$ Answer: 858 remainder 2 Showing remainder as a fraction: $858 \text{ r} 2$ $3 2^2 5^1 7^2 6$ Answer: $858 \frac{2}{3}$	per:	6 people won £8724 on the lottery. They spent £650 on a party to celebrate then shared the rest. How much did they each receive? How many divisions can you create which leave a remainder of 4/5, 2/3etc?

Year 5	Concrete	Pictorial	Conceptual		Using and applying
HTU ÷ TU (Using	558 ÷ 18 = Step 1: Identify a pair of factors for	the divisor e.g. $18 = 3 \times 6$	186	3 1	Try this with several numbers: choose a prime
factor pairs as divisors)	Step 2: Divide by one of the factors		$3 \overline{)5^{2}5^{1}8}$	6 1 8 6	number greater than 3, square it and divide the
_	Step 3: Divide the answer by the of	cher factor.	Answe	er: 558 ÷ 18 = 31	answer by 12. Look at the remainder. What do you
HTU ÷ TU (beginning	Using a tool kit of known facts to find <i>efficient</i> chunks. $558 \div 18 =$				notice? Why does this happen?
to look at chunking as	Tool kit - I know that:			4 Øl5 8	
a precursor to long	• 10 x 18 = 180 so		- <u>3 6 0</u> (20 x 18) 1 9 8		
division)	• 5 x 18 = • 20 x 18 =			- <u>180</u> (10 x 18)	
	• 100 x 18 =			- <u>18</u> (1 x 18)	
			A	0 nswer: 558 ÷ 18 = 31	

Year 6	Concrete	Pictorial	Conceptual	Using and applying
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication.	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c} 1735 \\ x 43 \\ 5205 \\ 244 \end{array} $ $ \begin{array}{c} 69400 \\ 242 \end{array} $ $ \begin{array}{c} (40 \times 1735) \\ 74605 \\ 1 \end{array} $ Answer: 1735 x 43 = 74605		Use similar examples (as above) adjusting to context as appropriate.
Divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.	432 ÷ 15 becomes 2 8 r 12 1 5 4 3 2 3 0 0 1 3 2 1 2 0 1 2 Answer: 28 remainder 12	432 ÷ 15 becomes 2 8 1 5 4 3 2 3 0 0 1 3 2 1 2 0 1 2 15×8 1 2 Answer: $28\frac{4}{5}$	432 ÷ 15 becomes 2 8 · 8 1 5 4 3 2 · 0 3 0 \	There are 432 guests at a wedding. Each table at dinner seats 15 people. How many tables are needed? A farmer had 450 eggs. 18 smashed so he put the rest into boxes of 15. How many boxes did he use? How many divisions can you create which result in a recurring decimal? Can you find a pattern in the numbers you used? Choose a 4 digit number and investigate fractional and decimal remainders when you divide by 9. What patterns do you notice?